

Sec. 6.3 Vertical Stretches and Compressions

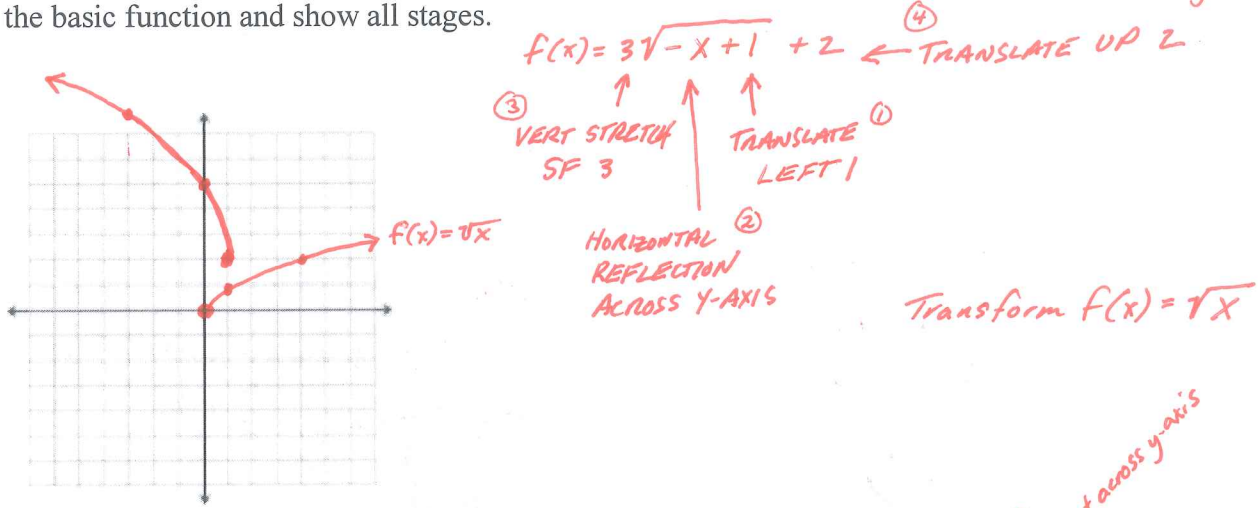
1. Compressions and Stretches

- a. **Vertical Stretch** – if $y = f(x)$ becomes $y = a f(x)$ where $a > 1$
- b. **Vertical Compression** – if $y = f(x)$ becomes $y = a f(x)$ for $0 < a < 1$

Ex. What would the equation be if the graph of $y = x^3$ is transformed by the following:

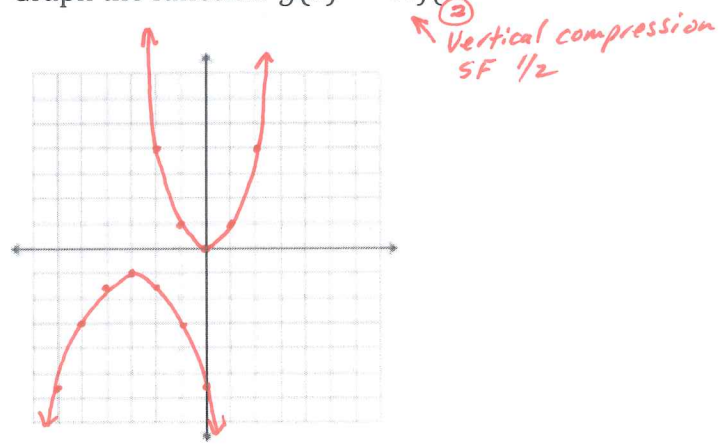
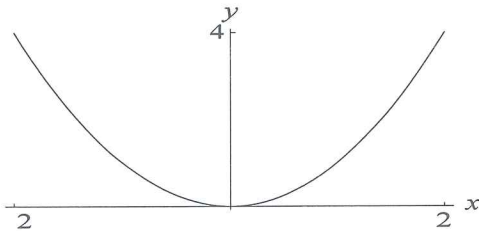
- a. Shift left 2 units and reflect about the x – axis: $y = -(x+2)^3$
- b. Shift down 2 units, reflect about the y – axis and stretched vertically by 4: $y = 4(-x)^3 - 2$

Ex. Graph the function $f(x) = 3\sqrt{1-x} + 2$ using transformations. Start with the graph of the basic function and show all stages.



Ex. The function $y = f(x)$ has the graph shown: Graph the function $g(x) = -\frac{1}{2}f(x+3) - 1$.

① Translate left 3 and down 1



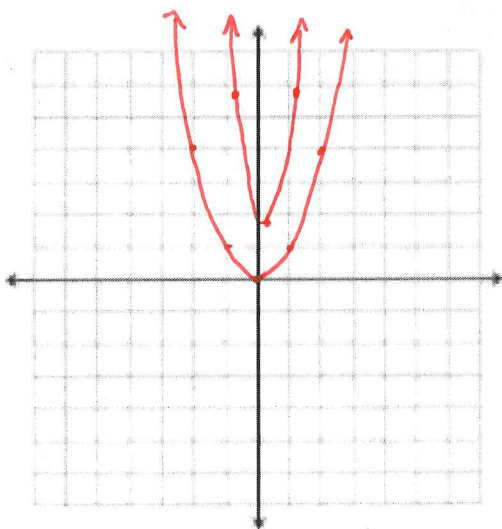
Stretch Factors and Average Rates of Change:

If $g(x) = k \cdot f(x)$, then on any interval, Average rate of change of $g = k \cdot$ (Average rate of change of f).

Ex. The function $s(t)$ gives the distance (miles) in terms of time (hours). If the average rate of change of $s(t)$ on $0 \leq t \leq 4$ is 70 mph, what is the average rate of change of $\frac{1}{2}s(t)$ on this interval?

$$\frac{1}{2}(70) = 35 \text{ mph}$$

Ex. Find the vertical stretch of the graph $f(x) = 4x^2 - 2x + 2$ by hand through completing the square. Find the vertex, intercepts, and graph by hand from the original quadratic function.



$$f(x) = 4 \left(x^2 - \frac{1}{2}x \right) + 2$$
$$= 4 \left(x^2 - \frac{1}{2}x + \frac{1}{16} \right) + 2 - \frac{1}{4}$$

$$f(x) = 4 \left(x - \frac{1}{4} \right)^2 + 1 \frac{3}{4}$$

↑ ↑ ↗
VERTICAL TRANSLATE
STRETCH RIGHT $\frac{1}{4}$ AND
SF 4 UP $1 \frac{3}{4}$

$$-\frac{1}{2} \cdot \frac{1}{2} = \left(-\frac{1}{4} \right)^2 = \frac{1}{16}$$
$$\frac{1}{16} \cdot 4 = \frac{4}{16} = \frac{1}{4}$$